10.1 Sequences and Series

Sequences

A *sequence* is an infinite list of terms:

$$a_1, a_2, a_3, \ldots, a_n, \ldots$$

Explicit definition

A sequence can be *explicitly* defined by giving a formula for the nth term a_n.

Find the first four terms:

$a_n = \frac{n^2-1}{n^2+1}$ $\qquad$ $a_n = (n-1)(n-2)(n-3)$ $\qquad$ $a_n = (-1)^n n^2$

Find an explicit definition of a_n:

$2, 4, 6, 8, 10, \ldots$ $\qquad$ $4, 6, 8, 10, 12, \ldots$ $\qquad$ $3, 9, 27, 81, \ldots$

$-2, 6, -18, 54, \ldots$ $\qquad$ $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \ldots$ $\qquad$ $1 \cdot 2,\ 2 \cdot 3, 3 \cdot 4, 4 \cdot 5, \ldots$

Recursive definition

A sequence can be "*recursively*" defined by giving the first term(s) and a rule for producing the remaining terms one at a time.

Find the first 4 terms:

$\begin{cases} a_1 = 16 \\ a_{n+1} = \sqrt{a_n} \end{cases}$ $\qquad$ $\begin{cases} a_1 = 4 \\ a_{n+1} = 1 + \frac{4}{a_n} \end{cases}$ $\qquad$ $\begin{cases} a_1 = 2 \\ a_2 = 3 \\ a_{n+1} = a_n - a_{n-1} \end{cases}$

(Infinite) Series and Partial Sums

Given a sequence $a_1, a_2, a_3, a_4, \dots$, we can sum the terms to obtain a *series*:

$$S_\infty = a_1 + a_2 + a_3 + a_4 + \cdots.$$

The sum of the first n terms of a series is called the nth partial sum of the series, denoted

$$S_n = a_1 + a_2 + a_3 + \cdots + a_n.$$

Find the partial sum $S_4 = a_1 + a_2 + a_3 + a_4$ of the sequence $1, -3, 5, -7, 9, -11, \dots$.

Find the sum of the series $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots$.

Summation Notation

Evaluate the following sums:

$\sum_{k=1}^{3} \ln k$ $\qquad$ $\sum_{i=3}^{5} (-1)^{i+1} 3i$

Write in summation notation:

$3 + 6 + 9 + 12$ $\qquad$ $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots$

10.2 Arithmetic Sequences and Series

Arithmetic Sequences

A sequence is *arithmetic* if the difference between successive terms is always the same. This difference d is called the *common difference*.

Recursive definition: $\begin{cases} a_1 = ? \\ a_{n+1} = a_n + d \end{cases}$

Explicit definition: $a_n = a_1 + d(n-1)$

Find a recursive definition and an explicit definition for the sequence $9, 5, 1, -3, \dots$.

Find the 12th term of the arithmetic sequence $2, 6, 10, \dots$

Arithmetic Series

The series associated to an arithmetic sequence is called an *arithmetic series*.

Partial sum: $S_n = n\left(\frac{a_1+a_n}{2}\right)$ **Series:** S_∞ is undefined (value of the sum is infinite)

Find the value of the following partial sums using the formula:

$9 + 5 + 1 + (-3)$ $1 + 2 + 3 + 4 + \cdots + 100$ $\sum_{k=1}^{10}(2k + 3)$

10.3 Geometric Sequences

Geometric Sequences

A sequence is *geometric* if the ratio between successive terms is always the same. This ratio is called the *common ratio*.

Recursive definition: $\begin{cases} a_1 = ? \\ a_{n+1} = a_n r \end{cases}$ **Explicit definition:** $a_n = a_1 r^{n-1}$

Find a recursive definition and an explicit definition for the sequence $18, -6, 2, -\frac{2}{3}, \ldots$.

Geometric Series

The series associated to a geometric sequence is called a *geometric series*.

Partial sum: $S_n = \frac{a_1(1-r^n)}{1-r}$, for $r \neq 1$ **Series:** $S_\infty = \frac{a_1}{1-r}$, for $|r| < 1$

Find the value of the partial sum and the series using the formulas above:

$3 + 6 + 12 + 24$ $100 - 10 + 1 - \frac{1}{10} + \cdots$ $\sum_{k=0}^{\infty} 3 \cdot 2^k$ $\sum_{k=1}^{\infty} 2 \cdot 3^{-k}$

Find fraction notation for the infinitely repeating decimal $8.\overline{19} = 8.191919\ldots$.

Arithmetic and Geometric Sequences and Series

	Arithmetic	**Geometric**
Sequences		
Recursive definition	$\begin{cases} a_1 = ? \\ a_{n+1} = a_n + d \end{cases}$	$\begin{cases} a_1 = ? \\ a_{n+1} = a_n r \end{cases}$
Explicit definition	$a_n = a_1 + d(n-1)$	$a_n = a_1 r^{n-1}$

Series		
Partial Sum	$S_n = a_1 + a_2 + \cdots + a_n$ $= \sum_{k=1}^{n} a_k$ $= n\left(\frac{a_1 + a_n}{2}\right)$	$S_n = a_1 + a_2 + \cdots + a_n$ $= \sum_{k=1}^{n} a_k$ $= a_1\left(\frac{1-r^n}{1-r}\right)$, for $r \neq 1$
(Infinite) Sum	$S_\infty = a_1 + a_2 + \cdots + a_n + \cdots$ $= \sum_{k=1}^{\infty} a_k$ is undefined!	$S_\infty = a_1 + a_2 + \cdots + a_n + \cdots$ $= \sum_{k=1}^{\infty} a_k$ $= \frac{a_1}{1-r}$, for $\lvert r \rvert < 1$