Elastic Collisions in 1D and 2D

Ch. 9.10-9.11
Elastic Collisions (1D)

Elastic Collision – KE of system conserved
Of course momentum is conserved too

\[
\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2
\]

\[
m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}
\]
Elastic Collisions (1D) - Analyzed

In general, 6 quantities, 2 equations
Can solve for any 2 quantities if other 4 known

For example:

\[ v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i} \]

\[ v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i} \]

Special cases: \( v_{2i} = 0 \) and \( m_1 = m_2, \ m_1 \gg m_2, \ m_1 \ll m_2 \)
(Signs are meaningful)
Elastic Collisions (2D)

KE of system conserved
Momentum in 2-D conserved

\[ KE: \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \]

\[ x \text{ mom: } m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} \cos \theta_1 + m_2 v_{2f} \cos \theta_2 \]

\[ y \text{ mom: } 0 = -m_1 v_{1f} \sin \theta_1 + m_2 v_{2f} \sin \theta_2 \]
Sample Problems

- A body of mass 2.0 kg makes an elastic collision with another body at rest and continues to move in the original direction but with one-fourth of its original speed. 
  (a) What is the mass of the other body? 
  (b) What is the speed of the two-body center of mass if the initial speed of the 2.0-kg body was 4.0 m/s? (#59, pg. 234)

- A proton with a speed of 500 m/s collides elastically with another proton initially at rest. The projectile and target protons then move along perpendicular paths, with the projectile path at 60° from the original direction. After the collision, what are the speeds of 
  (a) the target proton and 
  (b) the projectile proton? 
  (c) Confirm that the collision is, indeed, elastic. (#67, pg. 235)
A billiard ball moving at a speed of 2.2 m/s strikes an identical stationary ball a glancing blow. After the collision, one ball is found to be moving at a speed of 1.1 m/s in a direction making a 60° angle with the original line of motion.

(a) Find the velocity of the other ball.
(b) Can the collision be inelastic, given these data?

(#131, page 240)