Generalize Description of Work to Three Dimensions & Non-Constant Forces

\[ W = \vec{F} \cdot \vec{d} = \left| \vec{F} \right| \left| \vec{d} \right| \cos \theta \]

\[ = (F_x \hat{i} + F_y \hat{j} + F_z \hat{k}) \cdot (\Delta x \hat{i} + \Delta y \hat{j} + \Delta z \hat{k}) \]

\[ \{ \vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k} \]
\[ \vec{d} = \Delta x \hat{i} + \Delta y \hat{j} + \Delta z \hat{k} \]

Therefore, work is the scalar dot product given by:

\[ W = F_x \Delta x + F_y \Delta y + F_z \Delta z \]

Generalizes to Three Dimensions

(assumes force is constant for \( \Delta \)'s)

If force varies with position we can write above as

\[ dW = F_x \, dx + F_y \, dy + F_z \, dz \]

so that

\[ W = \int_{x_1}^{x_2} F_x \, dx + \int_{y_1}^{y_2} F_y \, dy + \int_{z_1}^{z_2} F_z \, dz \]

Generalizes to Non-Constant Forces
We emphasize these **two additional points**: 

**Point 1**  
**Work equals the area under the force-distance graph.**  

Here is an example from the text (page 154). The graph is magnitude of force versus displacement of the needle tip in a typical epidural procedure.  

What is the work done by the force exerted on the needle to get the needle to the epidural space at $x = 30$ mm?
Question 6 at back of chapter

Rank according to work done by force going from \( x = 0 \) to \( x = x_1 \), from most positive work to most negative work.
The graph shows the $x$ component $F_x$ of force that acts on a particle that begins at rest at $x = 0$.

(a) When is KE the greatest?
(b) When is speed the greatest?
(c) What is the direction of travel after reaching 6 m?
(d) How would answers change if speed were not zero at $x = 0$?
**Point 2**

The **spring force** is a special case that is important because many systems behave similarly.

Spring force $\rightarrow$ Hooke’s Law: $F = -kx$

One dimension situation

$$W = \int_{x_1}^{x_2} F_x \, dx = \int_{x_1}^{x_2} (-kx) \, dx = -\frac{1}{2} k x^2 \bigg|_{x_1}^{x_2} = \frac{1}{2} k x_1^2 - \frac{1}{2} k x_2^2$$
Sample Problem (Ch. 7, no. 29)

The only force acting on a 2.0 kg body as it moves along a positive $x$ axis has an $x$ component $F_x = -6x$ N, with $x$ in meters. The velocity of the body at $x = 3.0$ m is 8.0 m/s.

(a) What is the velocity of the body at $x = 4.0$ m?

(b) At what positive value of $x$ will the body have a velocity of 5.0 m/s?