Motion in Two and Three Dimensions

Analytic Approaches to Describing Motion

Ch. 4.1-4.4
Preview of Today’s Class

• Generalized Descriptions of Motion in Two and Three Dimensions

• Sample Problems
Generalized Description of Displacement

1-D

Displacement: \[ \vec{x} = x \hat{i} \]

Change in Displacement: \[ \vec{x}_2 - \vec{x}_1 = \Delta \vec{x} \]

3-D

Displacement: \[ \vec{r} = x \hat{i} + y \hat{j} + z \hat{k} \]

Change in Displacement: \[ \Delta \vec{r} = \vec{r}_2 - \vec{r}_1 \]

= \[ (x_2 - x_1) \hat{i} + (y_2 - y_1) \hat{j} + (z_2 - z_1) \hat{k} \]

= \[ \Delta x \hat{i} + \Delta y \hat{j} + \Delta z \hat{k} \]

To be sure, \( x \), \( y \), and \( z \) can all be functions of time \( \rightarrow x(t), y(t), \) and \( z(t) \)
Generalized Description of Velocity

Average velocity

\[ \vec{v}_{ave} = \frac{\vec{x}_2 - \vec{x}_1}{\Delta t} = \frac{\Delta \vec{x}}{\Delta t} \]

Instantaneous velocity

\[ \vec{v} = \frac{d\vec{x}}{dt} \]

1-D

Since \( x, y, \) and \( z \) can all be functions of time \( \rightarrow x(t), y(t), \) and \( z(t) \)

3-D

\[ \vec{v}_{ave} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\Delta x}{\Delta t} \hat{i} + \frac{\Delta y}{\Delta t} \hat{j} + \frac{\Delta z}{\Delta t} \hat{k} \]

\[ \vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt} \hat{i} + \frac{dy}{dt} \hat{j} + \frac{dz}{dt} \hat{k} \]
With SI units assumed

\[ x = -0.3t^2 + 7.2t + 28 \]

\[ y = 0.22t^2 - 9.1t + 30 \]

What is \( \vec{r} \) at \( t = 10 \) s?

What is \( \vec{v}_{\text{ave}} \) from \( t = 5 \) s to \( t = 15 \) s?

What is \( \vec{v} \) at \( t = 10 \) s?

What direction is the rabbit going at \( t = 10 \) s (express as an angle from the x-axis)?

What is \( \vec{a} \) at \( t = 10 \) s?
Unbelievable!
# Generalized Description of Acceleration

<table>
<thead>
<tr>
<th></th>
<th>1-D</th>
<th>3-D</th>
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</thead>
<tbody>
<tr>
<td><strong>Average acceleration</strong></td>
<td>$\vec{a}_{\text{ave}} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{\Delta \vec{v}}{\Delta t}$</td>
<td>$\vec{a}_{\text{ave}} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\Delta v_x}{\Delta t} \hat{i} + \frac{\Delta v_y}{\Delta t} \hat{j} + \frac{\Delta v_z}{\Delta t} \hat{k}$</td>
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<td><strong>Instantaneous acceleration</strong></td>
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Sample Problem 1

The displacement of a particle in m is given by

\[ \vec{r} = 2.00t^2\hat{i} - 6.00t\hat{j} + 2.00\hat{k} \]

if \( t \) is in seconds

(a) What is the velocity at \( t = 3.00 \) s in unit vector notation?
(b) What is the velocity at \( t = 3.00 \) s in magnitude and angle from an axis notation?
(c) What is the average velocity from \( t = 0 \) to 3.00 s?
(d) What is the average velocity from \( t = 1 \) to 4.00 s?
(e) What is the acceleration at \( t = 3.00 \) s?
(f) What is the average acceleration during the first 3.00 s?
Sample Problem 2

(a) When the displacement of a particle (in m) is given by

$$\vec{r} = 2.00t^2\hat{i} - 6.00t\hat{j} + 2.00\hat{k}$$

with \(t\) in seconds, at what time will the speed be zero? Describe the motion at \(t = 1.5\) s.

(b) When the displacement of a particle (in m) is given by

$$\vec{r} = (2.00t^2 - 6.00t)\hat{i} - 12.00\hat{j} + 2.00\hat{k}$$

with \(t\) in seconds, at what time will the speed be zero?