Uniform Accelerated Motion and Kinematic Equations

Ch. 2.5-2.8
Preview of Session

• Acceleration Defined
• Sample Problem
• Equations of Motion
• Sample Problems
• Graphical Representations

Very cool!
Acceleration Defined

Previously defined average velocity

\[ v_{\text{ave}} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} \]

and instantaneous velocity

\[ v = \frac{dx}{dt} \]

Similarly, average acceleration

\[ a_{\text{ave}} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} \]

and instantaneous acceleration

\[ a = \frac{dv}{dt} = \frac{d}{dt} \left( \frac{dx}{dt} \right) = \frac{d^2 x}{dt^2} \]
Sample Problem

The position of particle is given by \( x = 20.0t - 5.0t^3 \), where \( x \) is in meters and \( t \) is in seconds.

(a) What are the position, velocity, and acceleration of this particle at \( t = 3.0 \) s?

(b) What is the average velocity between \( 2.0 \) s and \( 4.0 \) s?

(c) What is the maximum positive coordinate reached?

(d) What is the maximum positive velocity?

(e) When is acceleration zero?
Sample Problem – Visualizing Solutions

\[ x = 20t - 5.0t^3 \]
Sample Problem – Visualizing Solutions

- Distance vs. Time:
  \[ x = 20t - 5.0t^3 \]

- Velocity vs. Time:
  \[ \mathbf{v} = 20 - 15.0t^2 \]

- Acceleration vs. Time:
  \[ \mathbf{a} = -30.0t \]
Kinematics Equations (1-2)

\[ a = a_{ave} = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t - t_0} \]

\[ v = v_0 + at, \text{ assumes } t_0 = 0 \]

\[ v_{ave} = \frac{x - x_0}{t} = \frac{v_0 + v}{2} \]

\[ \frac{x - x_0}{t} = \frac{v_0 + (v_0 + at)}{2} \quad \text{Using result above} \]

\[ x = x_0 + v_0 t + \frac{1}{2} at^2 \]
Kinematic Equations (3-5)

- Solve previous two for $t$ and set equal. Do the algebra to get:

\[ v^2 = v_0^2 + 2a(x - x_0) \]

- \[ x - x_0 = \frac{1}{2} (v_0 + v)t \]

- \[ x - x_0 = vt - \frac{1}{2}at^2 \]
Sample Problems – Set 1

• The speed of a bullet is measured to be 640 m/s as the bullet emerges from a barrel of length 1.20 m. Assuming constant acceleration, find the time that the bullet spends in the barrel after it is fired.

• The brakes on your car are capable of creating a deceleration of 5.2 m/s². If you are going 137 km/hr (85 mi/hr) and suddenly see a police officer, what is the minimum time you can get your car under the 97 km/hr (60 mi/h) speed limit?
Sample Problem - Set 2

A car moving with constant acceleration covered the distance between two points 60.0 m apart in 6.00 s. Its speed as it passes the second point was 15.0 m/s.

(a) What was the speed at the first point?
(b) What was the acceleration?
(c) At what prior distance from the first point was the car at rest?
(d) Graph $x$ versus $t$ and $v$ versus $t$ for the car, from rest ($t = 0$).
Review Graphical Connections

Simple case of one constant acceleration
Review Graphical Connections

Complex case – acceleration varies