5.4 Radians, Arc Length, and Angular Speed

The **radian measure** of an angle is the length of the arc on the unit circle cut out by the angle. In other words, the radian measure of an angle is the distance traveled by a point as it travels along the unit circle from the point \((1,0)\) on the initial side to the point where the terminal side intersects the unit circle.

**Converting between degree measure and radian measure**

Since the circumference of the unit circle is \(2\pi\), the angle with degree measure \(360^\circ\) has radian measure \(2\pi\). Similarly, a straight angle with degree measure \(180^\circ\) has radian measure \(\pi\). We can think of the fraction

\[
\frac{180^\circ}{\pi \text{ radians}}
\]

as being equal to the number 1, since we are dividing the measure of an angle by itself, just expressed in different units (as with 1 ft/12 in).

- To convert from **degrees** to **radians**, we multiply by “1” in the form of \(\frac{\pi \text{ radians}}{180^\circ}\).
- To convert from **radians** to **degrees**, we multiply go “1” in the form of \(\frac{180^\circ}{\pi \text{ radians}}\).

**Arc length and radian measure**

For a given angle in standard position, the ratio of the arc length \(s\) cut out by the angle on a circle (with center at the origin) to the radius \(r\) of the circle is always the same. In fact, it is equal to the radian measure \(\theta\) of the angle:

\[
\theta = \frac{s}{r}
\]

Put another way, \(s = r\theta\).

**Linear speed and angular speed**

**Linear speed** \(v\) is distance traveled per unit time. **Angular speed** \(\omega\) (omega) is amount of rotation per unit time. If we divide both sides of the preceding equation by time \(t\), we obtain:

\[
\frac{s}{t} = r \cdot \frac{\theta}{t}
\]

Thus \(v = s/t\) and \(\omega = \theta/t\), so:

\[
v = r\omega
\]